Fields of the Contrawound Toroidal Helix Antenna

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***Abstract*—Exact vector potential integrals are written in spherical coordinates for the CWTHA. From these the far fields are expressed, both for the CWTHA (dipole field) and for a single winding (loop field). The vector potentials are numerically integrated; the results give the dipole field for a given current and ratio of dipole field to loop field.**

***Index Terms*—Dipole, electrically small antenna, loop, toroidal helix.**

I. Introduction

A TOROIDAL helix consists of a helical coil formed into a toroidal shape and fed where the ends of the helix come together. Such an antenna has a circumferential current that produces a loop field and a toroidal tube of magnetic flux produced by the currents in the turns of the helix. As this circumferential magnetic field is similar in some respects to that of a dipole (on the toroid axis), this field is called the dipole field. It would be expected that the loop field would be much stronger than the dipole field. A clever idea was to wind two helical windings, in opposite directions, in the toroidal shape, with the windings excited π out of phase, see Fig. 1. Circumferential currents now cancel, reducing the loop field to zero, while the circumferential magnetic fields add, thus, augmenting the dipole field by a factor of 2. This concept was patented by Dr. James F. Corum in 1986 and 1988 [1], [2], and is here called the CWTHA. The toroid would be horizontal, with its axis vertical and the dipole field might act like a vertical dipole or whip.

Of most interest is an omnidirectional pattern in the plane of the toroid; this implies a near constant current around each winding. Moment method studies, which will be reported later, have indicated a winding length of roughly $\lambda/4$ or less to satisfy this condition. To effectuate the CWTHA concept, a substantial number of turns must be used; thus, the toroid diameter will be small in wavelengths. Winding length is approximately

$$\ell \simeq 2\pi a\sqrt{1+\left(\frac{N}{a/b}\right)^2} \tag{1}$$

where the toroid radius is a, the turn radius is b, and N is the number of turns.

It is the purpose of this paper to derive exact vector potentials for the CWTHA and to make far-field approximations for the dipole mode and loop mode fields.

Although the paper will show that the CWTHA is a very poor antenna, presentation of a detailed negative result is important due to the wide publicity engendered about the CWTHA and because many government organizations and companies have hoped that the CWTHA would replace whip-type antennas.

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II. Vector Potential Integrals

It is feasible to write exact vector potential integrals because the coordinates of a point on the helix can be written in terms of a single spherical coordinate variable ϕ [3]

$$\begin{aligned} x &= (a+b\cos N\phi)\cos\phi \\ y &= (a+b\cos N\phi)\sin\phi \\ z &= b\sin N\phi. \end{aligned} \tag{2}$$

The coordinate system is shown in Fig. 1. For the second winding the z coordinate changes sign.

The vector potential integral is

$$\underline{A} = \frac{1}{4\pi}\int I\frac{\exp -jkR}{R}\,\underline{d\ell} \tag{3}$$

where $k = 2\pi/\lambda$, and

$$r^2 = x^2+y^2+z^2 = a^2+b^2+2abC_n \tag{4}$$

with

$$C_n = \cos N\phi, \quad S_n = \sin N\phi.$$

The integration vector is

$$\underline{d\ell} = \underline{a}_r\,dr + \underline{a}_\theta r\,d\theta + \underline{a}_\phi r\sin\theta\,d\phi. \tag{5}$$

The vector potentials of interest are expressed in spherical coordinates for one winding

$$A_r = \frac{Io}{4\pi}\int_{-\pi}^{\pi}\underline{d\ell}\cdot\underline{a}_R\frac{\exp -jkR}{R}\,d\phi \tag{6}$$

$$A_\theta = \frac{Io}{4\pi}\int_{-\pi}^{\pi}\underline{d\ell}\cdot\underline{a}_{\theta 0}\frac{\exp -jkR}{R}\,d\phi \tag{7}$$

$$A_\phi = \frac{Io}{4\pi}\int_{-\pi}^{\pi}\underline{d\ell}\cdot\underline{a}_{\phi 0}\frac{\exp -jkR}{R}\,d\phi. \tag{8}$$

Note that this problem is unusual in that all three components are needed to get the exact fields. The constant value of current is I_0 and R is the distance from the integration point to the far-field point x_0, y_0, z_0.

Dot products relate the integration vector to the far-field vector [4]

$$\underline{a}_R\cdot\underline{a}_r = \sin\theta_0\sin\theta\cos(\phi-\phi_0)+\cos\theta_0\cos\theta \tag{9}$$

$$\underline{a}_{\theta 0}\cdot\underline{a}_r = \cos\theta_0\sin\theta\cos(\phi-\phi_0)-\sin\theta_0\sin\theta \tag{10}$$

$$\underline{a}_{\phi 0}\cdot\underline{a}_r = \sin\theta\sin(\phi-\phi_0) \tag{11}$$

$$\underline{a}_R\cdot\underline{a}_\theta = \sin\theta_0\cos\theta\cos(\phi-\phi_0)-\cos\theta_0\sin\theta \tag{12}$$

$$\underline{a}_{\theta 0}\cdot\underline{a}_\theta = \cos\theta_0\cos\theta\cos(\phi-\phi_0)+\sin\theta_0\sin\theta \tag{13}$$

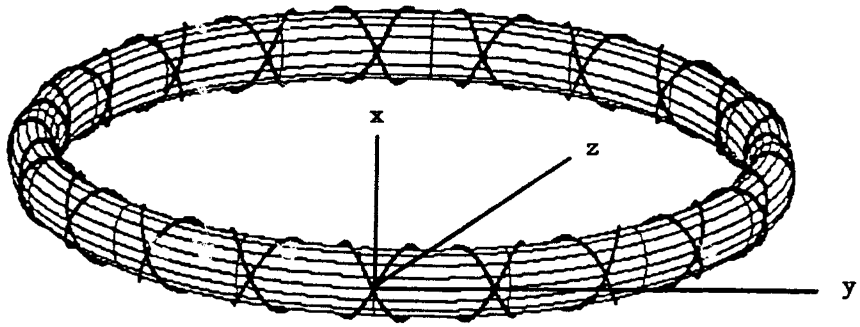

Fig. 1. Contrawound Toroidal Helix Antenna (on foam core).

$$\underline{a}_{\phi 0} \cdot \underline{a}_{\theta} = \cos\theta \sin(\phi - \phi_0) \tag{14}$$
$$\underline{a}_{R} \cdot \underline{a}_{\phi} = -\sin\theta_0 \sin(\phi - \phi_0) \tag{15}$$
$$\underline{a}_{\theta 0} \cdot \underline{a}_{\phi} = -\cos\theta_0 \sin(\phi - \phi_0) \tag{16}$$
$$\underline{a}_{\phi 0} \cdot \underline{a}_{\phi} = \cos(\phi - \phi_0). \tag{17}$$

Equation (6) is expanded by use of (9), (12), and (15) with (5). Similarly (7) uses (10), (13), and (16) with (5); (8) uses (11), (14), and (17) with (5). From (2), θ can be written in terms of ϕ

$$\tan\theta = (a + bC_n)/bS_n. \tag{18}$$

Trig functions and derivatives needed in (9) through (17) are simply

$$\sin\theta = (a + bC_n)/r, \quad \cos\theta = bS_n/r \tag{19}$$
$$\frac{dr}{d\phi} = -\frac{NabS_n}{r} \tag{20}$$
$$\frac{d\theta}{d\phi} = -\frac{Nb(aC_n + b)}{r^2}. \tag{21}$$

Finally the distance is

$$R^2 = R_0^2 + a^2 + b^2 + 2abC_n - 2R_0[(a + bC_n) \sin\theta_0 \cos(\phi - \phi_0) + bS_n \cos\theta_0]. \tag{22}$$

For the phase, the far-field approximation gives

$$R \simeq R_0 - (a + bC_n)\sin\theta_0 \cos(\phi - \phi_0) - bS_n \cos\theta_0. \tag{23}$$

In the denominator, $R = R_0$ as usual.

Electric field is found from the vector potential

$$\underline{E} = -j\eta/k \,\mathrm{curl}\,\mathrm{curl}\,\underline{A} \tag{24}$$

where $\eta \simeq 120\pi$. This formulation (Lorentz) avoids the scalar potential [5].

The far-field vector potential integrals now become

$$A_R = \frac{-I_0 \exp -jkR_0}{4\pi R_0} \int_{-\pi}^{\pi} [NbS_n \sin\theta_0 \cos(\phi - \phi_0) - NbC_n \cos\theta_0 - (a + bC_n)\sin\theta_0 \sin(\phi - \phi_0)] \times \exp jkA \, d\phi \tag{25}$$

$$A_{\theta 0} = \frac{-I_0 \exp -jkR_0}{4\pi R_0} \int_{-\pi}^{\pi} [NbC_n \sin\theta_0 + NbS_n \cos\theta_0 \times \cos(\phi - \phi_0) + (a + bC_n)\cos\theta_0 \sin(\phi - \phi_0)] \times \exp jkA \, d\phi \tag{26}$$

$$A_{\phi 0} = \frac{I_0 \exp -jkR_0}{4\pi R_0} \int_{-\pi}^{\pi} [(a + bC_n)\cos(\phi - \phi_0) - NbS_n \sin(\phi - \phi_0)] \exp jkA \, d\phi \tag{27}$$

where

$$A = (a + bC_n)\sin\theta_0 \cos(\phi - \phi_0) + bS_n \cos\theta_0. \tag{28}$$

Note that the derivatives in (23) and (24) produce $k^2 r_0$, so that

$$E_\theta \simeq 120\pi k A_\theta \quad \text{and} \quad E_\phi \simeq 120\pi k A_\phi. \tag{29}$$

E_r has not been calculated, as E_θ $(\theta = 90)$ is of most interest.

For these small antennas, the loop field reduces to the textbook result

$$E_\phi \simeq \frac{30\pi k a I_0 \exp -jkR_0}{R_0} J_1(ka \sin\theta) \tag{30}$$

and this reduces further

$$E_\phi \simeq \frac{30\pi k^2 a^2 I_0 \sin\theta \exp -jkR_0}{R_0}. \tag{31}$$

III. Double Winding

When the counter winding is added, $z, \cos\theta$ and $d\theta/d\phi$ change sign. With the 180° excitation of this winding, several terms in the vector potential integrals cancel. In A_R, only the $NbC_n \cos\theta_0$ term remains. In $A_{\theta 0}$, only the $NbC_n \sin\theta_0$ term remains. Both terms in $A_{\phi 0}$ cancel, leaving zero as expected. All remaining terms are doubled.

IV. Numerical Evaluation

Neither of the vector potential integrals can be integrated in closed form. Evaluation was performed via double precision complex numerical integration. A 128 point Gaussian and a 300 point Simpson gave the same results to at least six significant figures.

An approximate formula is obtained from [5]; for $\theta_0 = 90°$, the E_θ expression (5) of that paper can be summed exactly, with the result

$$E_\theta \simeq 15\pi I_0 N k a k^2 b^2 / R_0. \tag{32}$$

In azimuth the pattern is constant (omnidirectional), even for small N. Table I shows results computed from eqs. (26) and

(29), and from the ring-bar result eq. (32). Agreement is excellent. This validates the E_θ vector potential results.

Table II gives the ratio of E_θ/E_ϕ for several CWTHA cases. This field ratio is very closely

$$\frac{E_\theta}{E_\phi} \simeq \frac{Nk^2b^2}{2ka}. \tag{31}$$

An obvious question is what parameters give the highest E_θ value. Using the constraint of maximum winding length equal $\lambda/4$, the field is maximized when kb is maximized. This occurs for $a/b = 2$. Next ka is maximized, which occurs for small N. However this yields maximum dipole field for a given current, but since the radiation resistance increases as N^2, a small value of N is a poor choice.

The E_θ/E_ϕ field ratio behaves differently. Again, kb is maximized; next the number of turns is selected large, as increasing ka also increases the loop field. Because the objective is to cancel the loop mode field, this ratio is less important than the value of E_θ alone.

For the contrawound toroidal helix, where E_θ has only a single term in the vector potential, the numerically integrated results are exactly twice those of Table I. Because the second winding doubles the $E_{\theta 0}$ component, the E_θ/E_ϕ ratios of Table II are doubled, or $+6$ dB added. However the ratios are still small: -32 to -50 dB.

V. Measurements

A measurement program utilizing several facilities is nearly complete and will be reported in a subsequent paper along with detailed moment method results. Both measurements and simulation have proven difficult due to the very small values of radiation resistance. Almost all measured data of which the authors are aware are contaminated by feed cable radiation and by chamber/range background levels. As mentioned earlier, the toroid diameter is small in wavelengths. For example, with a toroid/turn diameter ratio of 5 and 10 turns, the toroid diameter in wavelengths must be no larger than $.036\lambda$. The loop field radiation resistance is of the order of 30 milliohms so the loop efficiency is not high. Further, the large VSWR occurring when reactance matched to 50 ohms results in a significant matching circuit loss. If the antenna is not matched, the mismatch loss is of the order of 40 dB. This is relevant as the measured dipole field must be compared with another measured field and the loop field is convenient. Absolute gain measurement appears impossible due to microohm radiation resistance for the CWTHA.

Low as the unmatched loop field gain is, the CWTHA gain is much lower. For the example above, the dipole field radiation resistance is roughly 16 microohms and the unmatched mismatch loss is roughly 75 dB; values very difficult to measure.

In moment method simulation, representing each turn by twelve linear segments gives results close to circular values. Again, for the example above, with turn diameter of 0.0072λ, segment lengths are roughly 0.002λ. Some popular codes produce significant errors in calculating impedance of such short skew segments.

TABLE I
EXACT AND RING-BAR E_θ

a/λ	b/λ	N	E_θ exact	E_θ ring-bar [6]
.01	.001	10	.1168E-2	.1169E-2
.02	.001	10	.2333E-2	.2338E-2
.01	.002	10	.4673E-2	.4676E-2
.01	.001	20	.2337E-2	.2338E-2

TABLE II
DIPOLE/LOOP FIELD RATIO

a/λ	b/λ	N	E_θ/E_ϕ	E_θ/E_ϕ
.01	.001	10	.313E-2	-50.1 dB
.02	.001	10	.157E-2	-56.1 dB
.01	.002	10	.123E-1	-38.2 dB
.01	.001	20	.625E-2	-44.1 dB

VI. Directivity and Bandwidth

The CWTHA, both single winding and dual winding, have the directivity of an electrically small antenna: 1.5. Gain, as defined by IEEE, includes efficiency and efficiency is typically low. Radiation resistance for a single winding (loop field) is essentially that of a single turn loop so that efficiencies are generally in the range of 40 to 80%, depending on the wire diameter. The CWTHA radiation resistances tend to be in the milliohm to microohm range so that efficiencies, hence gains, are very small.

The CWTHA is basically an inductive winding, with reactances of the order of several hundred ohms. Because the total antenna resistance tends to be well below one ohm, the Q tends to be large, and the bandwidth much less than one percent. It was stated by the inventor (Dr. Corum) that this was a narrowband antenna.

Only a few patterns have been calculated as they proved to be as expected, those for an electrically short dipole (in elevation). Azimuth patterns are closely circular.

VII. Conclusion

The omnidirectional CWTHA is a very poor radiator of dipole fields, as shown by the exact vector potential analysis. Results are in excellent agreement with a previous ring-bar approximate analysis.

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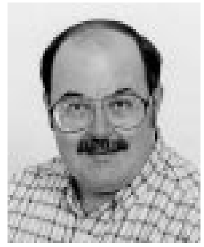

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